Logical functors and connectives

Logic and argumentation techniques
Akos Gyarmathy
Negation: $\neg$

- The function of the negation is to reverse the truth value of a given propositions (sentence).
- If A is true, then $\neg A$ is false. If A is false then $\neg A$ is true.
- In natural language the equivalent of negation is: *it is not the case that X.* or *it is false that X.* where X is a proposition.
It is raining.
The negation:
It is not true that it is raining.

I love ice-cream.
The negation:
It is not the case that I love ice-cream.
Conjunction: $\wedge$

- $\wedge$
- Single propositions: $P$, $Q$
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Conjunction: $P \wedge Q$
  - An action is good when it makes people happy, and keeping your promises is always good.
  - Conjunctions of $P \wedge Q$ are $P$, $Q$. 
How to analyze conjunction?

Analysis of conjunctions

1. Ice cream is tasty and I love my dog.
2. I have a heart and a kidney.
3. I am married and I have two kids.

– What are the conjuncts?
– When is it true?
– Truth table and definition of the conjunction.

Definition of conjunction

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P &amp; Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Conjunctions in arguments

• Conjunction introduction
  1. P
  2. Q
  Therefore P \& Q

• Conjunction elimination
  1. P\&Q
  Therefore P
  1. P\&Q
  Therefore Q
Problematic examples for $\wedge$

- Justice and tolerance are valuable.
- Colleen and Errol got married.
- I went out and had dinner.
- One false move and I shoot.
- He was tired but he wanted to keep going.
Disjunction: \( v \)

- \( v \)
- Single propositions: \( P, Q \)
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Disjunction: \( P \lor Q \)
  - Either an action is good when it makes people happy, or keeping your promises is always good.
  - *Disjuncts* of \( P \lor Q \) are \( P, Q \).
How to analyze disjunction?

• Blue is a color or $7+3=10$.
• I am a teacher or I am a man.
• You are students or you are young.
• It is raining now or it is Friday.
  – What are the disjuncts?
  – When is it true?
  – Truth table and definition of the disjunction.
Problematic disjunctions

• Either an action is good when it makes people happy, or keeping your promises is always good.
• Socrates is dead or Socrates is alive.
• A proposition is either true or false.
Inclusive and exclusive disjunction

**Inclusive disjunction**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P ∨ Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

**Exclusive disjunction**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P ⊕ Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Find the difference inclusive or exclusive

• You either pass or fail this course.
• A person is either tall or blonde.
• It is either raining or it is not.
• One hour is exactly 55 minutes or 24 hours is 1440 minutes.
• America was discovered by Colombus or Americo Vespucci.
Disjunction types in arguments

- **Valid inference:** Disjunctive syllogism.
  1. $P \oplus Q$
  2. $\neg P$
  Therefore $Q$.

- **A formal fallacy!**
  **Affirming the disjunct**
  1. $PvQ$
  2. $P$
  Therefore $\neg Q$.
Conditionals: $\supset$

- $\supset$
- Single propositions: $P, Q$
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Conditional: $P \supset Q$: if $P$ then $Q$.
  - If an action is good when it makes people happy then keeping your promises is always good.
  - Antecedent: $P$
  - Consequence: $Q$
Problematic cases for conditionals

- Which one is the antecedent?
- If $P$ then $Q$
- If $R, S$
- $D$ if $E$
- $Z$ only if $F$
- *If* $p$ *refers always to the antecedent*
The case of a biconditional $p \equiv q$

- $p \equiv q$
- An action is good when it makes people happy if and only if keeping your promises is always good.
- IFF
- $p \equiv q$ is the same as (If $p$ then $q$) and (If $q$ then $p$).
Definition of a conditional?

Conditional: $P \supset Q$

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>$P \supset Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

$P \equiv Q$

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>$P \equiv Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Not ($P$ and not $Q$)

Not ($P \oplus Q$)