

Validity and reconstructing arguments

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Validity

Definition: $X \models A$ if and only if any evaluation satisfying everything in X also satisfies A .

- If the argument from X to A is valid then there is no evaluation making the premises X true and the conclusion A false. **($X \& A$) must be true!**

($X \& \neg A$) must be false!

- Put the premises of the argument and the *negation of the conclusion* in a list!!

Sometimes it is easy to check validity

1) When I go to the doctor I always wear black underwear.

2) I went to the doctor yesterday.

C: I wore black underwear.

1) When I go to the doctor I always wear black underwear.

2) I went to the bank yesterday.

C: I wore black underwear.

1) When I go to the doctor I always wear black underwear.

2) I did not wear black underwear yesterday.

C: I did not go to the doctor yesterday

1) When I go to the doctor I always wear black underwear.

2) I wore black underwear yesterday.

C: I went to the doctor yesterday.

1) When I go to doctor I go to the bank.

2) I wore black underwear yesterday.

C: I went to the bank yesterday

Put the premises of the argument and the *negation of the conclusion* in a list

If the propositions cannot be true together, **the argument is valid**;

- If they can be true together, **the argument is not valid**.

- A branch is **closed** when it contains a formula and its negation (the formula need not be atomic). A branch that is not closed is said to be open.

(1): $p \supset q$
(2): $r \vee \neg q$
C: $((p \vee q) \supset r)$



(1): $p \supset q$
(2): $r \vee \neg q$
 $\neg((p \vee q) \supset r)$

Analytic tables

(1): $p \supset q$
 (2): $r \vee \neg q$
 C: $((p \vee q) \supset r)$



(1): $p \supset q$
 (2): $r \vee \neg q$
 $\neg((p \vee q) \supset r)$

$p \supset q$
 $r \vee \neg q$
 $\neg((p \vee q) \supset r)$
 $|$
 $p \vee q$
 $\neg r$

Double negation

$\neg\neg A$
 $|$
 A

Conjunction

$A \& B$
 $|$
 A
 B

Negated conjunction

$\neg(A \& B)$
 $/ \quad \backslash$
 $\neg A \quad \neg B$

Disjunction

$A \vee B$
 $/ \quad \backslash$
 $A \quad B$

Negated disjunction

$\neg(A \vee B)$
 $|$
 $\sim A$
 $\sim B$

Error corrected

Conditional

$A \supset B$
 $/ \quad \backslash$
 $\neg A \quad B$

Negated conditional

$\neg(A \supset B)$
 $|$
 A
 $\sim B$

Error corrected

Biconditional

$A \equiv B$
 $/ \quad \backslash$
 $A \quad \neg A$
 $B \quad \neg B$

Negated biconditional

$\neg(A \equiv B)$
 $/ \quad \backslash$
 $A \quad \neg A$
 $\neg B \quad B$

Analytic tables

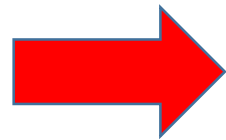
(1): $p \supset q$
(2): $r \vee \neg q$
C: $((p \vee q) \supset r)$



(1): $p \supset q$
(2): $r \vee \neg q$
 $((\neg p \vee q) \supset r)$

$p \supset q$
 $r \vee \neg q$
 $\neg((p \vee q) \supset r) \checkmark$
|
 $p \vee q$
 $\neg r$!

Analytic tables

$$\begin{array}{l} p \supset q \\ r \vee \neg q \\ \neg((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \neg r \end{array}$$

$$\begin{array}{l} p \supset q \\ r \vee \neg q \checkmark \\ \neg((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \neg r \\ \swarrow \quad \searrow \\ r \quad \neg q \\ \times \end{array}$$

(1): $p \supset q$
(2): $r \vee \neg q$
C: $((p \vee q) \supset r)$



(1): $p \supset q$
(2): $r \vee \neg q$
 $((\neg p \vee q) \supset r)$

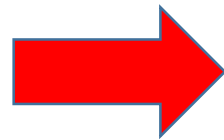
Analytic tables

$$\begin{array}{l} p \supset q \\ r \vee \sim q \\ \sim((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \sim r \end{array}$$

(1): $p \supset q$
(2): $r \vee \sim q$
C: $((p \vee q) \supset r)$



(1): $p \supset q$
(2): $r \vee \sim q$
 $((\sim p \vee q) \supset r)$

$$\begin{array}{l} p \supset q \\ r \vee \sim q \checkmark \\ \sim((p \vee q) \supset r) \checkmark \\ | \\ p \vee q \\ \textcircled{\sim r} \\ / \quad \backslash \\ \textcircled{r} \quad \sim q ! \\ \times \end{array}$$


Analytic tables

$$\begin{array}{l}
 p \supset q \\
 r \vee \sim q \\
 \sim((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q \\
 \sim r
 \end{array}$$

(1): $p \supset q$
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(1): $p \supset q$
 (2): $r \vee \sim q$
 $((\sim p \vee q) \supset r)$

$$\begin{array}{l}
 p \supset q \\
 r \vee \sim q \checkmark \\
 \sim((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q \\
 \textcircled{\sim r} \\
 \swarrow \quad \searrow \\
 \textcircled{r} \quad \sim q \\
 \times
 \end{array}$$


$$\begin{array}{l}
 p \supset q \checkmark \\
 r \vee \sim q \checkmark \\
 \sim((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q \\
 \sim r \\
 \swarrow \quad \searrow \\
 r \quad \sim q \\
 \times \quad \swarrow \quad \searrow \\
 \quad \sim p \quad q \\
 \quad \quad \times
 \end{array}$$

Analytic tables

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 p \supset q \\
 r \vee \sim q \\
 \neg((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q \\
 \sim r
 \end{array}$$

(1): $p \supset q$
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(1): $p \supset q$
 (2): $r \vee \sim q$
 $((\sim p \vee q) \supset r)$

$$\begin{array}{l}
 p \supset q \\
 r \vee \sim q \checkmark \\
 \neg((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q \\
 \textcircled{\sim r} \\
 \swarrow \quad \searrow \\
 \textcircled{r} \quad \sim q \\
 \times
 \end{array}$$

$$\begin{array}{l}
 p \supset q \checkmark \\
 r \vee \sim q \checkmark \\
 \neg((p \vee q) \supset r) \checkmark \\
 | \\
 p \vee q \\
 \textcircled{\sim r} \\
 \swarrow \quad \searrow \\
 \textcircled{r} \quad \textcircled{\sim q} \\
 \times \quad \swarrow \quad \searrow \\
 \sim p! \quad \textcircled{q} \\
 \times
 \end{array}$$


Analytic tables

(1): $p \supset q$
 (2): $r \vee \neg q$
 C: $((p \vee q) \supset r)$



(1): $p \supset q$
 (2): $r \vee \neg q$
 C: $((\neg p \vee q) \supset r)$

$p \supset q$
 $r \vee \neg q$
 $\neg((p \vee q) \supset r) \checkmark$
 |
 $p \vee q$
 $\neg r$

$p \supset q$
 $r \vee \neg q \checkmark$
 $\neg((p \vee q) \supset r) \checkmark$
 |
 $p \vee q$
 $\neg r$
 / \
 r $\neg q$
 \times

$p \supset q \checkmark$
 $r \vee \neg q \checkmark$
 $\neg((p \vee q) \supset r) \checkmark$
 |
 $p \vee q$
 $\neg r$
 / \
 r $\neg q$
 \times
 / \
 $\neg p$ q
 \times



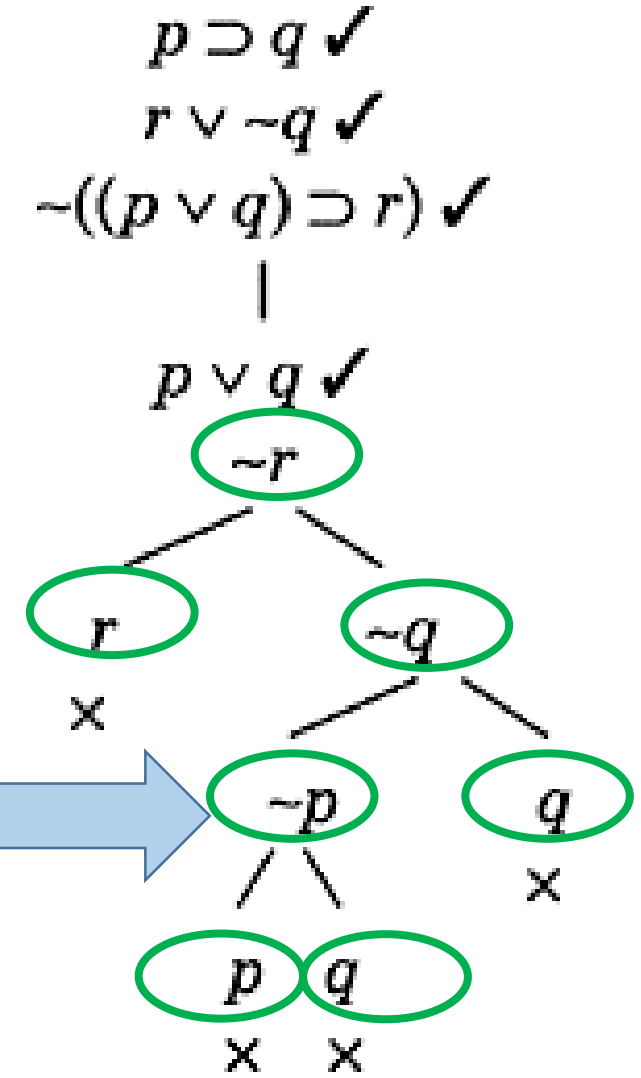
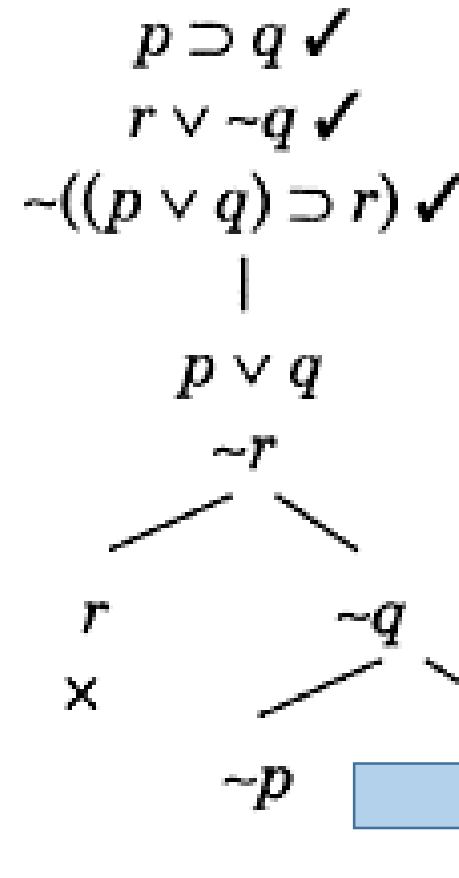
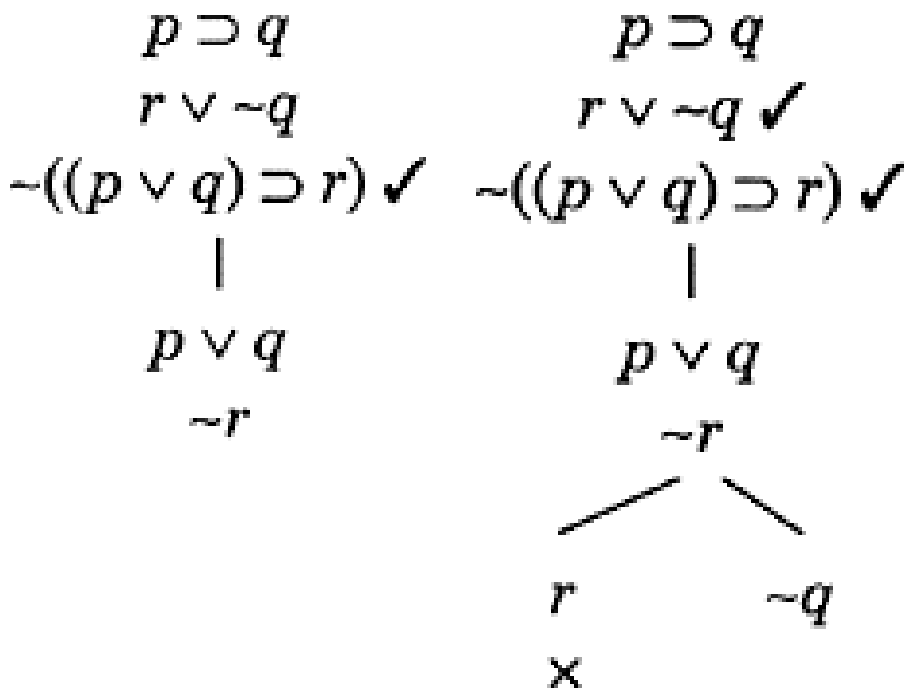
$p \supset q \checkmark$
 $r \vee \neg q \checkmark$
 $\neg((p \vee q) \supset r) \checkmark$
 |
 $p \vee q \checkmark$
 $\neg r$
 / \
 r $\neg q$
 \times
 / \
 $\neg p$ q
 / \
 p q
 \times \times

Analytic tables

(1): $p \supset q$
 (2): $r \vee \neg q$
 C: $((p \vee q) \supset r)$



(1): $p \supset q$
 (2): $r \vee \neg q$
 C: $((\neg p \vee q) \supset r)$



1) All cats are reptiles.
2) Bugs Bunny is a cat.
Therefore: Bugs Bunny is a reptile.

1) If Socrates was a philosopher, then he wasn't a historian.
2) Socrates wasn't a historian.
Therefore Socrates was a philosopher.

1) If P, then Q
2) P
Therefore, Q

1) If P, then Q
2) Not Q
Therefore P

1) If P then S
2) If Q then R
3) Not S or Not R
Therefore not P or not Q.

1) If P then Q
2) If Q then S
Therefore if P then S.

1) A or B
2) Not A
Therefore B

1) P or Q
2) If P then S
3) If Q then R
Therefore S or R.

Practice in groups! Create an argument to the form below. Is this argument valid?

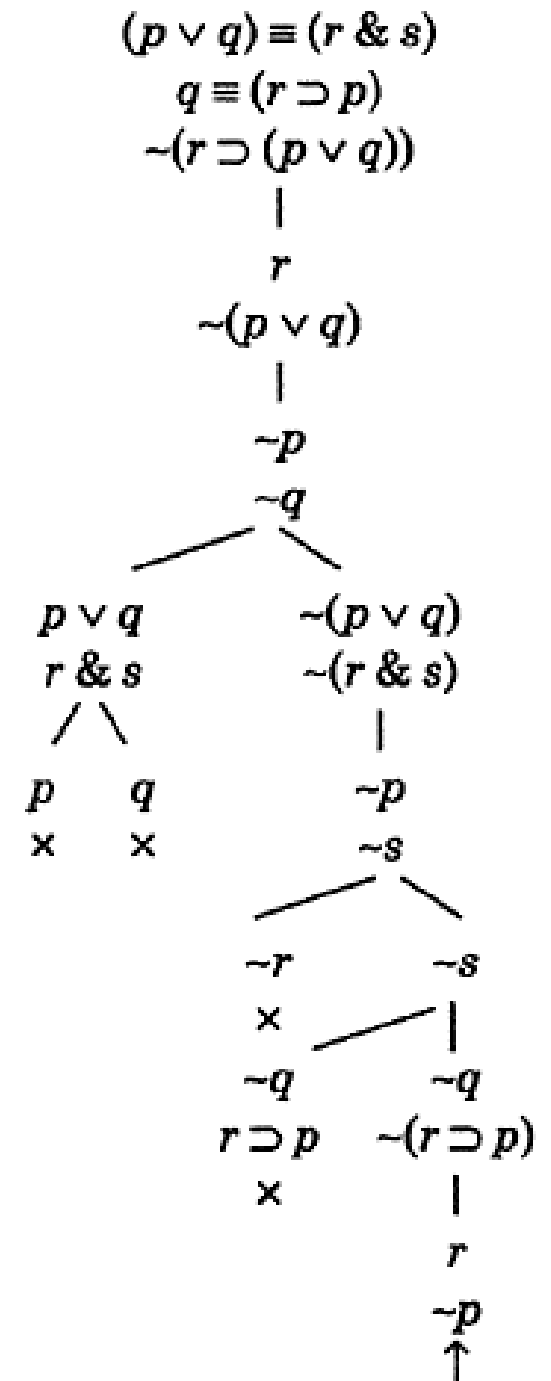
$$1) \quad p \vee q \equiv (r \wedge s)$$

$$2) \quad q \equiv (r \supset p)$$

$$\therefore r \supset (p \vee q)$$

Practice in groups! Create an argument to the form below. Is this argument valid?

- 1) $p \vee q \equiv (r \wedge s)$
 - 2) $q \equiv (r \supset p)$
- $\therefore r \supset (p \vee q)$

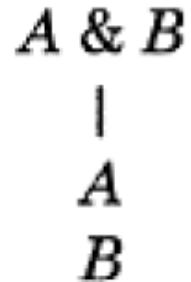


The rules of the analytic table

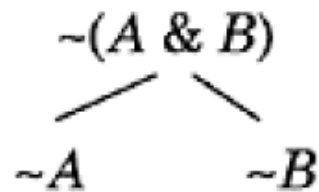
Double negation



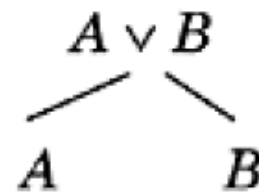
Conjunction



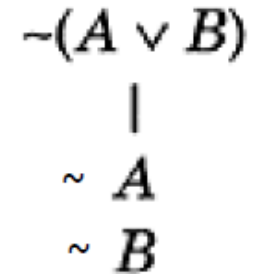
Negated conjunction



Disjunction

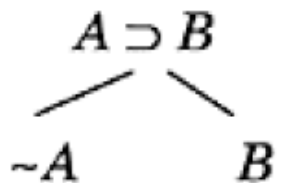


Negated disjunction

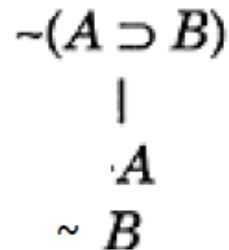


Error corrected

Conditional

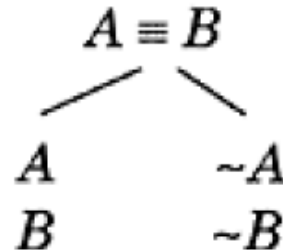


Negated conditional



Error corrected

Biconditional



Negated biconditional

