Logical functors and connectives

Logic and argumentation techniques

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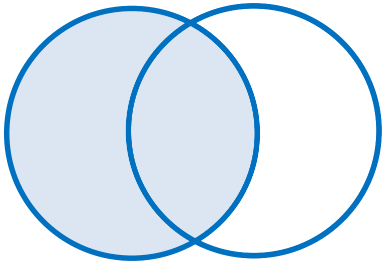
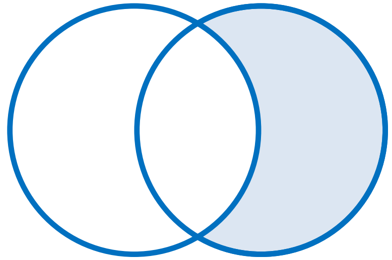
Negation: ¬

- The function of the negation is to reverse the truth value of a given propositions (sentence).
- If A is true, then ¬A is false. If A is false then ¬A is true.
- In natural language the equivalent of negation is: *it is not the case that* X. or *it is false that* X. where X is a proposition.



p



¬p

P	¬P
T	F
F	T

It is raining.
The negation:
It is not true that it is raining.

I love ice-cream.
The negation:
It is not the case that I love ice-cream.

Conjunction: ∧

- ∧
- Single propositions: P, Q
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Conjunction: P ∧ Q
  - An action is good when it makes people happy, and keeping your promises is always good.
  - Conjuncts of P ∧ Q are P, Q.

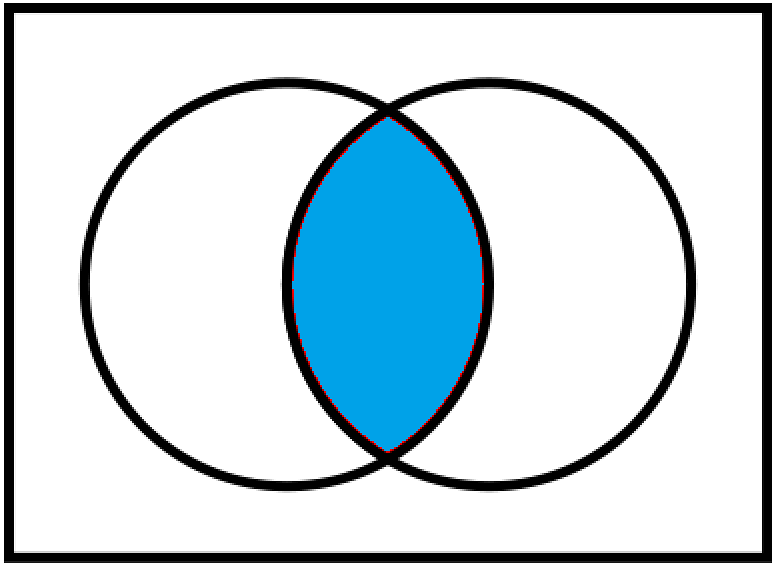
How to analyze conjunction?

Analysis of conjunctions

1. **Ice cream is tasty and I love my dog.**
2. **I have a heart and a kidney.**
3. **I am married and I have two kids.**
   – What are the conjuncts?
   – When is it true?
   – Truth table and definition of the conjunction.

Definition of conjunction

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F



Conjunctions in arguments

- Conjunction introduction

  1. P
  2. Q

  Therefore $P \wedge Q$

- Conjunction elimination

  1. $P \wedge Q$

  Therefore P

  1. $P \wedge Q$

  Therefore Q

Problematic examples for $\wedge$

- Justice and tolerance are valuable.
- Colleen and Errol got married.
- I went out and had dinner.
- One false move and I shoot.
- He was tired but he wanted to keep going.

Disjunction: v

- V
- Single propositions: P, Q
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Disjunction: P V Q
  - Either an action is good when it makes people happy, or keeping your promises is always good.
  - *Disjuncts* of P V Q are P, Q.

How to analyze disjunction?

- Blue is a color or 7+3=10.
- I am a teacher or I am a man.
- You are students or you are young.
- It is raining now or it is Friday.
  - What are the disjuncts?
  - When is it true?
  - Truth table and definition of the disjunction.

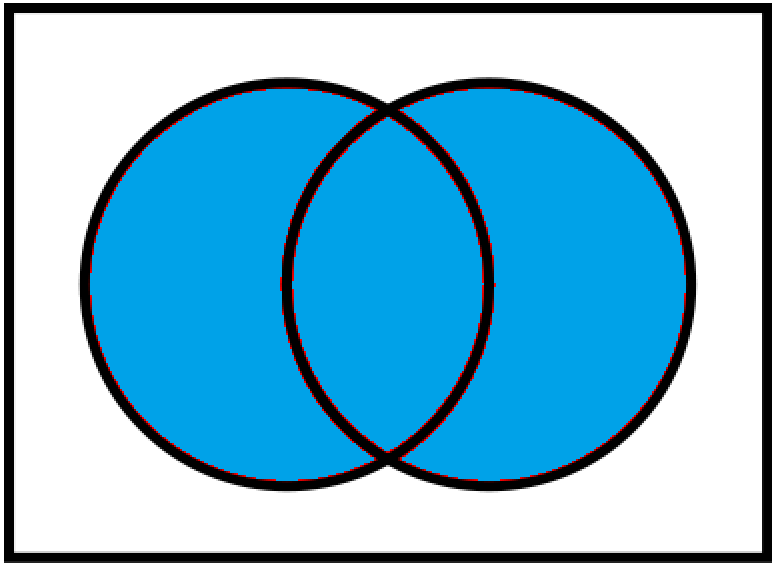
Problematic disjunctions

- Either an action is good when it makes people happy, or keeping your promises is always good.
- Socrates is dead or Socrates is alive.
- A proposition is either true or false.

Inclusive and exclusive disjuntion

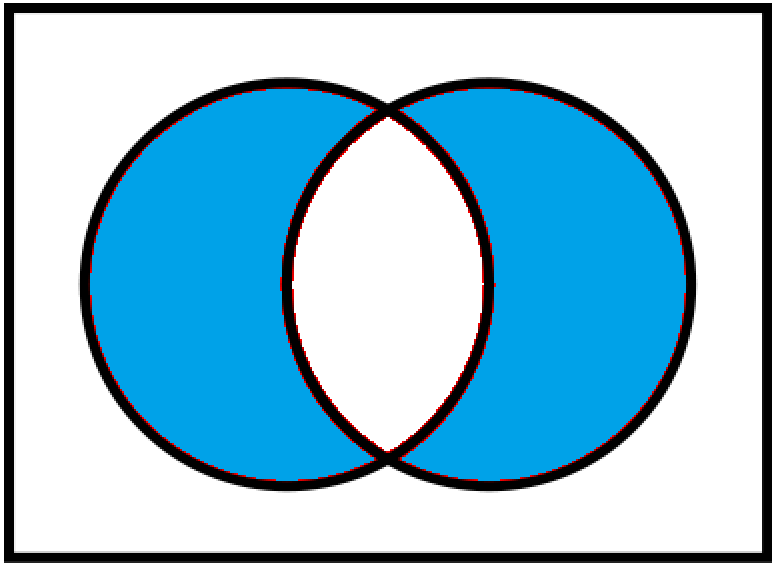
Inclusive disjunction

P	Q	PVQ
T	T	T
T	F	T
F	T	T
F	F	F



Exclusive disjunction

P	Q	P⊕Q
T	T	F
T	F	T
F	T	T
F	F	F



Find the difference inclusive or exclusive

- You either pass or fail this course.
- A person is either tall or blonde.
- It is either raining or it is not.
- One hour is exactly 55 minutes or 24 hours is 1440 minutes.
- America was discovered by Colombus or Americo Vespucci.

Disjunction types in arguments

- **Valid inference:** Disjunctive syllogism.
  1. $P \oplus Q$
  2. $\neg P$

  Therefore Q.

- **A formal fallacy! Affirming the disjunct**
  1. $P \vee Q$
  2. P

  Therefore $\neg Q$

Conditionals: $\supset$

- $\supset$
- Single propositions: P, Q
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Conditional: $P \supset Q$: if P then Q.
  - If an action is good when it makes people happy then keeping your promises is always good.
  - Antecedent: P
  - Consequence: Q

Problematic cases for conditionals

- Which one is the antecedent?
- If P then *Q*
- If R*, S*
- *D* if E
- *Z* only if F
- ***If p** refers always to the antecedent*

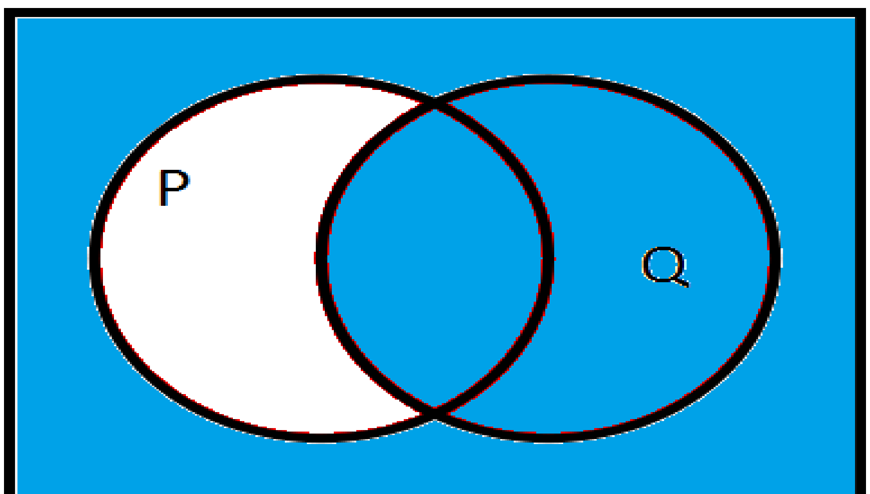
The case of a biconditional p≡q

- P≡Q
- An action is good when it makes people happy **if and only if** keeping your promises is always good.
- IFF
- P≡Q is the same as (If P then Q) and (If Q then P).

Definition of a conditional?

Conditional: P⊃Q

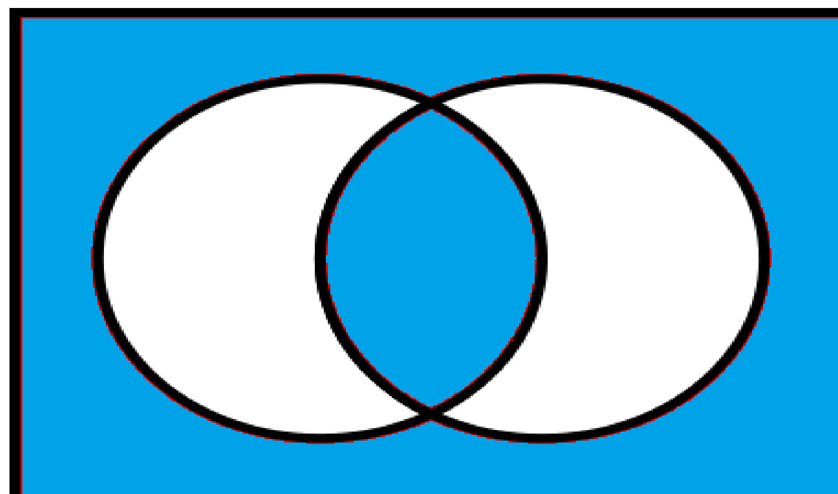
P	Q	P⊃Q
T	T	T
T	F	F
F	T	T
F	F	T



Not (P and not Q)

P≡Q

P	Q	P≡Q
T	T	T
T	F	F
F	T	F
F	F	T



Not (P⨁Q)