Introduction to research methodology

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Basic concepts of logic

- Deductive inference
- Inductive inference
- Proposition
- Argument
  - Premise
  - Conclusion
- Validity
- Soundness
- Logical function
- Connective
  - Conjunction
  - Disjunction
  - Conditional
  - Equivalence

Deduction and Induction

DEDUCTION

Criterion of adequacy:

A statement is a valid deductive consequence of a group of other statements if and only if it would be self-contradictory to assert all the statements in the group and to deny the statement of the conclusion.

Example

(1) "If you have a current password, then you can log on to the network"

(2) "You have a current password"

Therefore:

"You can log on to the network"

- Logically valid
  - Truth of the premises necessitate the truth of the conclusion
  - Deductive inference is truth-preservinga
- Deductive nomological laws
- Laws of court
- Laws of Nature

Deduction and **Induction**

INDUCTION
Criterion of Adequacy:
As evidence accumulates, the *degree* to which the collection of true evidence statements comes to *support* a hypothesis, as measured by the logic, should tend to indicate that false hypotheses are probably false and that true hypotheses are probably true.

Example: 62 percent of voters in a random sample of 400 registered voters (polled on February 20, 2004) said that they favor John Kerry over George W. Bush for President in the 2004 Presidential election. This supports with a probability of at least .95 the hypothesis that between 57 percent and 67 percent of all registered voters favor Kerry over Bush for President (at or around the time the poll was taken)

Inductive inference

- Is not logically valid
  - Premises do not necessitate the conclusion
- Inductive statistical laws
- Statistical probabilities

Propositions and Arguments

Arguments

Arguments in everyday situations take place between people.

Arguments give reasons for believing the truth of a proposition.

Logic studies the information content of these arguments.

An argument is a list of propositions, called the ***premises,*** followed by a word such as ‘therefore’, or ‘so’, and another proposition, called the ***conclusion***.
The ***reason for accepting*** the information in the conclusion is based on the premises.

If everything is determined, people are not free. Premise
People are free. Premise
So not everything is determined. Conclusion

What is a proposition?

- A proposition is a predicative sentence that only contains a subject and a predicate

S is P.

Logics of different "size"

- Propositional logic
  - Socrates is a man: P
  - WFFs refer to propositions.
- Predicate/First-order logic
  - Socrates is a man: ∀xFx
  - WFFs refer to propositions
  - Quantification over individuals
- Second order logic
  - Socrates is a man: ∀P∀x(x∈P)
  - WFFs refer to propositions
  - Quantification over individuals
  - Quantification over properties (set)

Predicate logic

- Contains names and predicates
  - Names: a proper name is a simple expression that serves to pick out an individual thing – 'Socrates'
    - a-o
  - a predicate is an expression that results in a sentence when a number of names are inserted in the appropriate places – …is a man.
    - F, G, etc…

Valid arguments

- An argument is valid if and only if whenever the premises are true, so is the conclusion.
- In other words, *it is impossible* for the premises to be true while at the same time the conclusion is false.

Sound arguments

- An argument is sound, just in the case where it is valid, and, in addition, the premises are
- all true. So, the conclusion of a sound argument must also be true.
- soundness appeals to the *truth* of the matter.

Valid Argument Scheme

All bats are birds. F

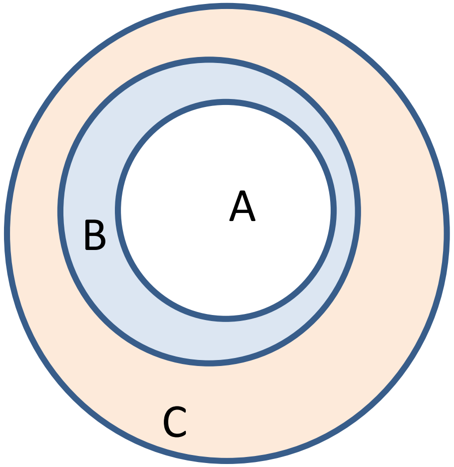
All birds fly. F Not sound

Therefore all bats fly. T

All whales are mammals. T

All mammals have hearts. T Sound

All whales have hearts. T



All A's are B's
All B's are C's

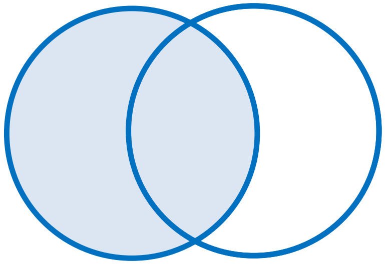
So all A's are C's

c

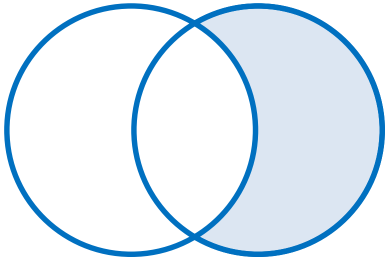
Logical functions: the connectives

Negation: ¬

- The function of the negation is to reverse the truth value of a given propositions (sentence).
- If A is true, then ¬A is false. If A is false then ¬A is true.
- In natural language the equivalent of negation is: *it is not the case that* X. or *it is false that* X. where X is a proposition.



p



¬p

P	¬P
T	F
F	T

It is raining.
The negation:
It is not true that it is raining.

I love ice-cream.
The negation:
It is not the case that I love ice-cream.

Conjunction: $\wedge$

- $\wedge$
- Single propositions: P, Q
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Conjunction: $P \wedge Q$
  - An action is good when it makes people happy, and keeping your promises is always good.
  - Conjuncts of $P \wedge Q$ are P, Q.

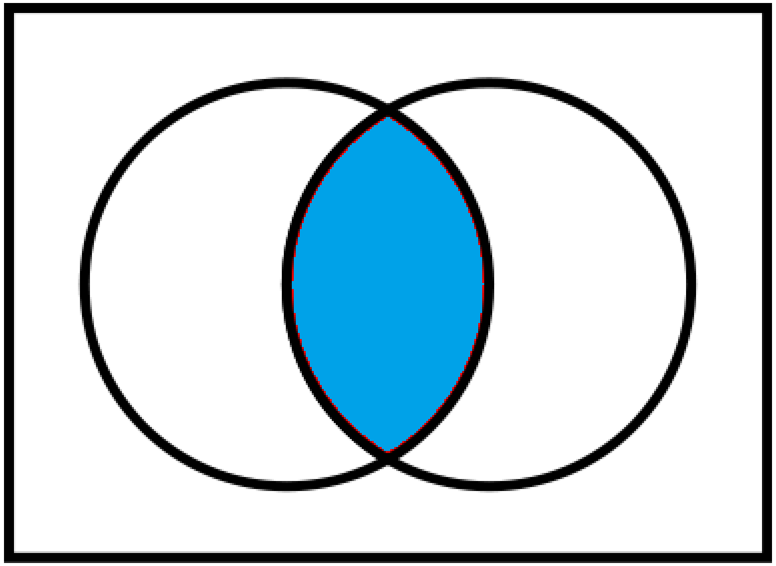
How to analyze conjunction?

Analysis of conjunctions

1. **Ice cream is tasty and I love my dog.**
2. **I have a heart and a kidney.**
3. **I am married and I have two kids.**
   - What are the conjuncts?
   - When is it true?
   - Truth table and definition of the conjunction.

Definition of conjunction

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F



Conjunctions in arguments

- Conjunction introduction
  1. P
  2. Q

  Therefore $P \land Q$
- Conjunction elimination
  1. $P \land Q$

  Therefore P
  1. $P \land Q$

  Therefore Q

Problematic examples for $\wedge$

- Justice and tolerance are valuable.
- Colleen and Errol got married.
- I went out and had dinner.
- One false move and I shoot.
- He was tired but he wanted to keep going.

Disjunction: v

- V
- Single propositions: P, Q
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Disjunction: P V Q
  - Either an action is good when it makes people happy, or keeping your promises is always good.
  - *Disjuncts* of P V Q are P, Q.

How to analyze disjunction?

- Blue is a color or 7+3=10.
- I am a teacher or I am a man.
- You are students or you are young.
- It is raining now or it is Friday.
  - What are the disjuncts?
  - When is it true?
  - Truth table and definition of the disjunction.

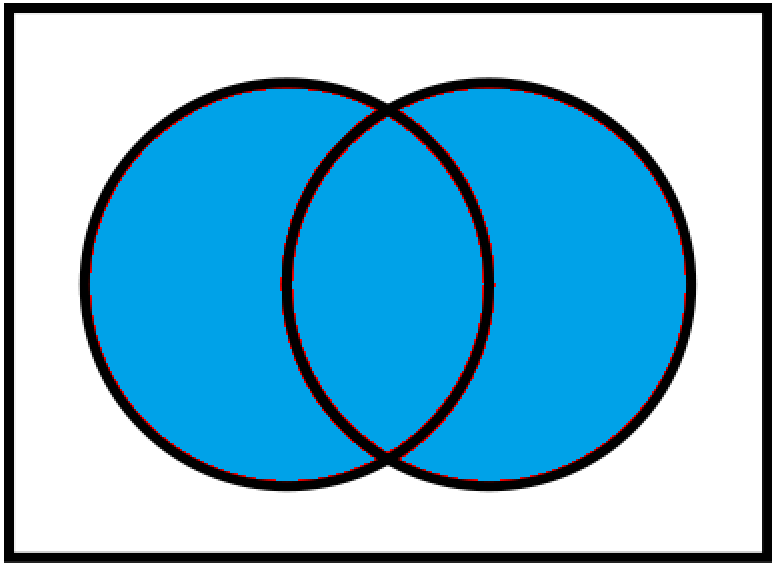
Problematic disjunctions

- Either an action is good when it makes people happy, or keeping your promises is always good.
- Socrates is dead or Socrates is alive.
- A proposition is either true or false.

Inclusive and exclusive disjuntion

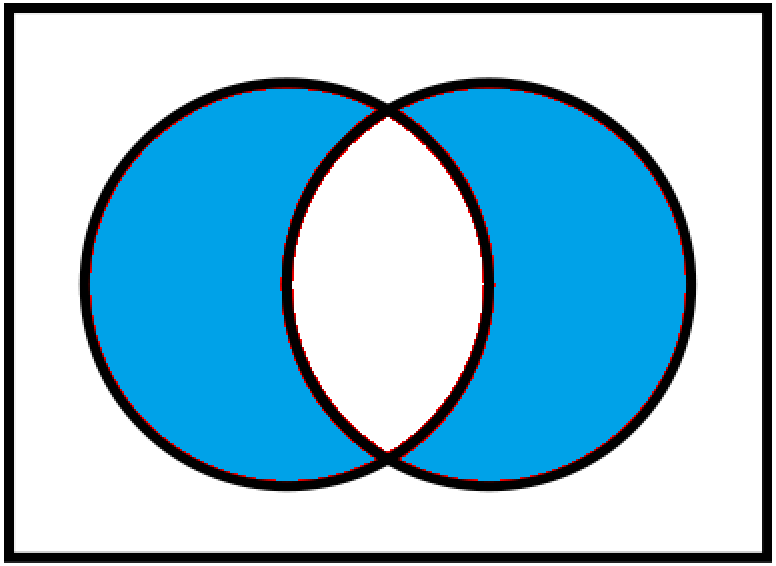
Inclusive disjunction

P	Q	PVQ
T	T	T
T	F	T
F	T	T
F	F	F



Exclusive disjunction

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F



Find the difference inclusive or exclusive

- You either pass or fail this course.
- A person is either tall or blonde.
- It is either raining or it is not.
- One hour is exactly 55 minutes or 24 hours is 1440 minutes.
- America was discovered by Colombus or Americo Vespucci.

Disjunction types in arguments

- **Valid inference:** Disjunctive syllogism.
  1. $P \oplus Q$
  2. $\neg P$

  Therefore Q.

- **A formal fallacy! Affirming the disjunct**
  1. PvQ
  2. P

  Therefore ¬Q

Conditionals: ⊃

- ⊃
- Single propositions: P, Q
  - An action is good when it makes people happy.
  - Keeping your promises is always good.
- Conditional: P ⊃ Q: if P then Q.
  - If an action is good when it makes people happy then keeping your promises is always good.
  - Antecedent: P
  - Consequence: Q

Problematic cases for conditionals

- Which one is the antecedent?
- If P then *Q*
- If R, *S*
- *D* if E
- *Z* only if F
- ***If p*** *refers always to the antecedent*

The case of a biconditional p≡q

- P≡Q
- An action is good when it makes people happy **if and only if** keeping your promises is always good.
- IFF
- P≡Q is the same as (If P then Q) and (If Q then P).

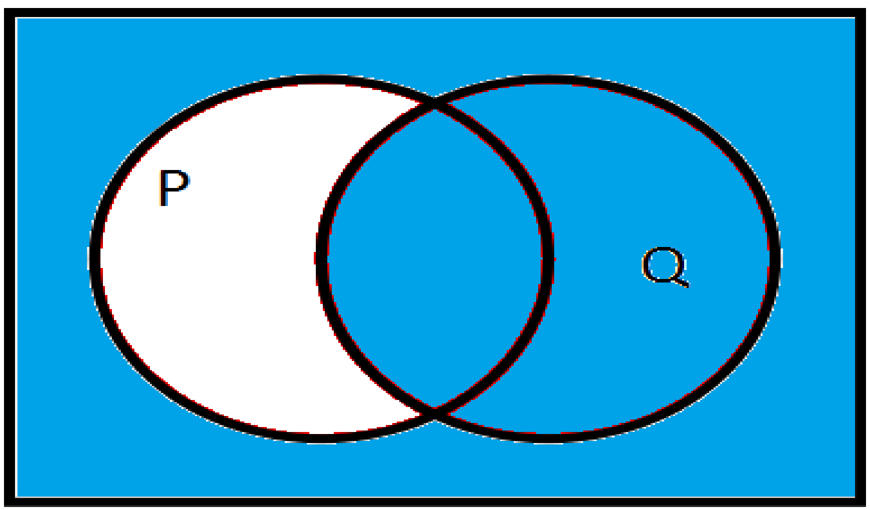
Definition of a conditional?

Conditional: P⊃Q

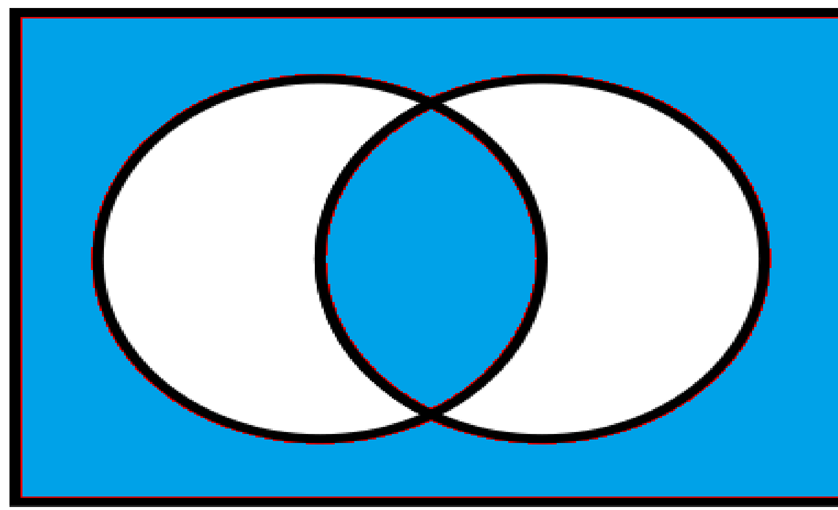
P	Q	P⊃Q
T	T	T
T	F	F
F	T	T
F	F	T

P≡Q

P	Q	P≡Q
T	T	T
T	F	F
F	T	F
F	F	T



Not (P and not Q)



Not (P⊕Q)

Few examples of complex transformations: De Morgan's laws

¬ (p ∧ q) ≡ ¬p v ¬q

P	Q	P ∧ Q	¬ (P ∧ Q)	¬ P	¬ Q	¬P V ¬Q
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

- ¬ (p v q) ≡ ¬p ∧ ¬q

P	Q	PVQ	¬ (P v Q)	¬ P	¬ Q	¬P ∧ ¬Q
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Demonstrating validity

- Definition: X $\models$ A if and only if any evaluation satisfying everything in X also satisfies A.
  - X: set of premises
  - A: conclusion
- If the argument from X to A is valid then there is no evaluation making the premises X true and the conclusion A false.
- (X&A) must be true!
- (X&¬A) must be false!