Deductive validity, possibilities and introduction to inductive logic

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Validity

Definition: $X \models A$ if and only if any evaluation satisfying everything in X also satisfies A.

If the argument from X to A is valid then there is no evaluation making the premises X true and the conclusion A false. **(X&A) must be true!**

(X&¬A) must be false!

Validity

Validity ≠ **Soundness**

If the premises are true then the conclusion cannot be false

Soundness: the argument is valid and its premises are true

In order to check validity it is necessary to check all the possible distributions of truth values of the combination of the premises and the conclusion

Validity: how complex is an array of truth values?

Most propositions have at least 2 possible truth values: True or False

P
T
F

The number of the possible combinations of truth values for a set of statements becomes twice as many with the addition of every single propositon.

P	Q	R
T	T	T
T	F	T
F	T	T
F	F	T
T	T	F
T	F	F
F	T	F
F	F	F

The truth values of complex propositions (that are compounds of more than one simple proposition) are evaluated on all possible combinations of the truth values of their compounds

P	Q	R	$P \wedge Q$	$(P \wedge Q) \supset R$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T
T	T	F	T	F
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

Validity check on all possible truth value distribution

Validity: If the argument from *X* (set of premises) to *A* (conclusion) is valid then there is no evaluation making the premises *X* true and the conclusion *A* false.
(X∧A) must be true!
(X∧¬A) must be false!

For any valid argument any distribution of truth values is impossible where the premises are true while in the same time the conclusion is false.

The following argument:
1) (P ∧ Q) ⊃ R
2) (P ∧ Q)
Therefore: R

Is **valid** if and only if **there is no** case when **((P ∧ Q) ⊃ R) and (P ∧ Q) are both true but R is false.**

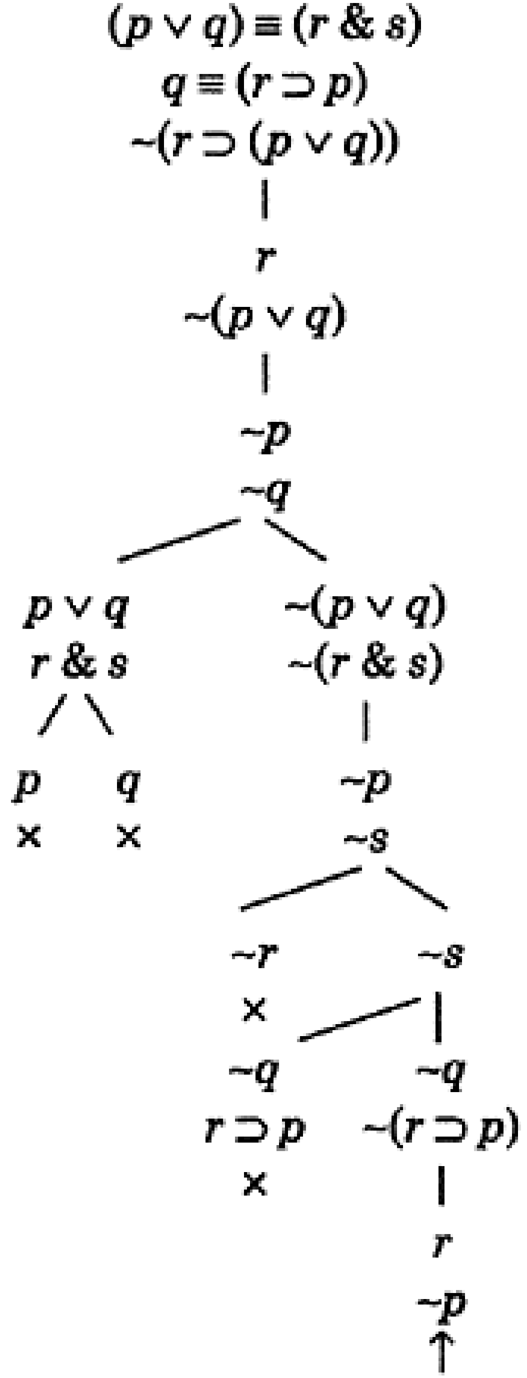
Since the first raw is the only possible case where all the premises are true together, it is sufficient for validity if the conclusion is also true there

P	Q	R	P ∧ Q	(P ∧ Q) ⊃ R	(P ∧ Q) ∧ ((P ∧ Q) ⊃ R)	((P ∧ Q) ∧ ((P ∧ Q) ⊃ R)) ∧ R
T	T	T	T	T	T	**T**
T	F	T	F	T	F	**F**
F	T	T	F	T	F	**F**
F	F	T	F	T	F	**F**
T	T	F	T	F	F	**F**
T	F	F	F	T	F	**F**
F	T	F	F	T	F	**F**
F	F	F	F	T	F	**F**

Practice in groups! Create an argument to the form below. Is this argument valid?

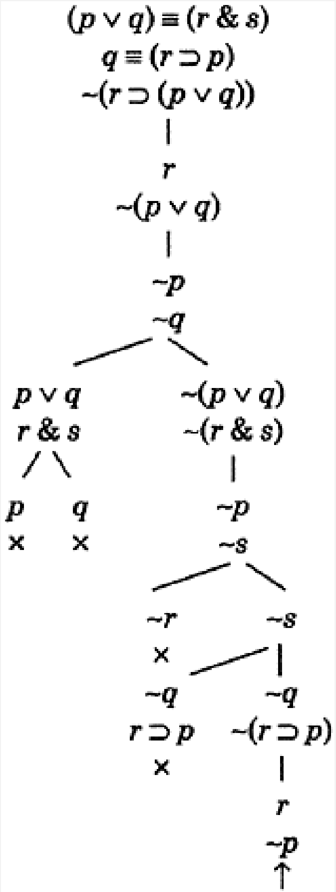
1) pvq ≡ (r ∧ s)
2) q ≡ (r ⊃ p)

∴ r ⊃ (p v q)



Different representations of a deductive validity

The tree structure of analytic tables



Truth tables

P	Q	R	P ∧ Q	(P ∧ Q) ⊃ R	(P ∧ Q) ∧ ((P ∧ Q) ⊃ R)	((P ∧ Q) ∧ ((P ∧ Q) ⊃ R)) ∧ R
T	T	T	T	T	T	**T**
T	F	T	F	T	F	**F**
F	T	T	F	T	F	**F**
F	F	T	F	T	F	**F**
T	T	F	T	F	F	**F**
T	F	F	F	T	F	**F**
F	T	F	F	T	F	**F**
F	F	F	F	T	F	**F**

Analytic tables: representing possible distributions of truth values on a tree structure

Put the **premises** and **the *negation of the conclusion*** in a list

- If the propositions cannot be true together, **the argument is valid**;
- If they can be true together, **the argument is invalid**.

(1): $p \supset q$
(2): $r \vee \neg q$
C: $((p \vee q) \supset r)$

(1): $p \supset q$
(2): $r \vee \neg q$
$\neg((p \vee q) \supset r)$

Analytic tables

(1): p⊃q
(2): rv¬q
C: ((pvq) ⊃r)

(1): p⊃q
(2): rv¬q
¬((pvq) ⊃r)

$p \supset q$

$r \vee \sim q$

$\sim((p \vee q) \supset r)$

$p \vee q$

$\sim r$

Double negation

$\sim\sim A$

A

Conjunction

$A \,\&\, B$

A

B

Negated conjunction

$\sim(A \,\&\, B)$

$\sim A$ | $\sim B$

Disjunction

$A \vee B$

A | B

Negated disjunction

$\sim(A \vee B)$

$\sim A$

$\sim B$

Error corrected

Conditional

$A \supset B$

$\sim A$ | B

Negated conditional

$\sim(A \supset B)$

A

$\sim B$

Error corrected

Biconditional

$A \equiv B$

A, B | $\sim A$, $\sim B$

Negated biconditional

$\sim(A \equiv B)$

A, $\sim B$ | $\sim A$, B

Analytic tables

(1): p⊃q
(2): rv¬q
C: ((pvq) ⊃r)

(1): p⊃q
(2): rv¬q
¬((¬pvq) ⊃r)

$$p \supset q$$
$$r \vee \sim q$$
$$\sim((p \vee q) \supset r) \checkmark$$
$$|$$
$$p \vee q$$
$$\sim r$$

!

Analytic tables

(1): p⊃q

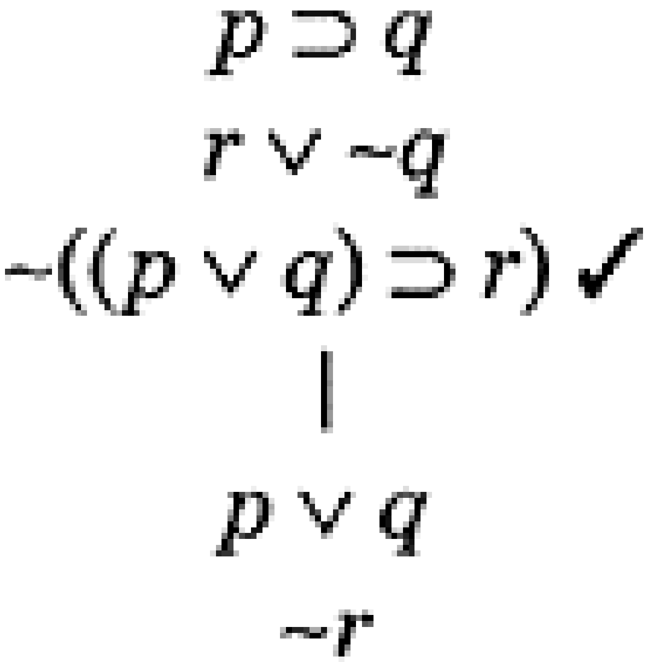
(2): rv¬q

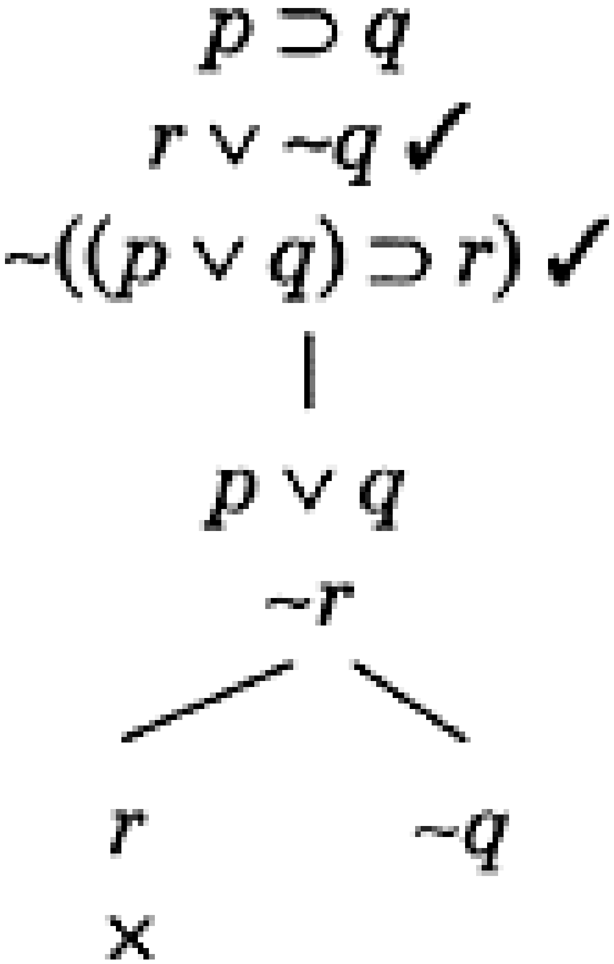
C: ((pvq) ⊃r)

(1): p⊃q

(2): rv¬q

¬((pvq) ⊃r)





Analytic tables

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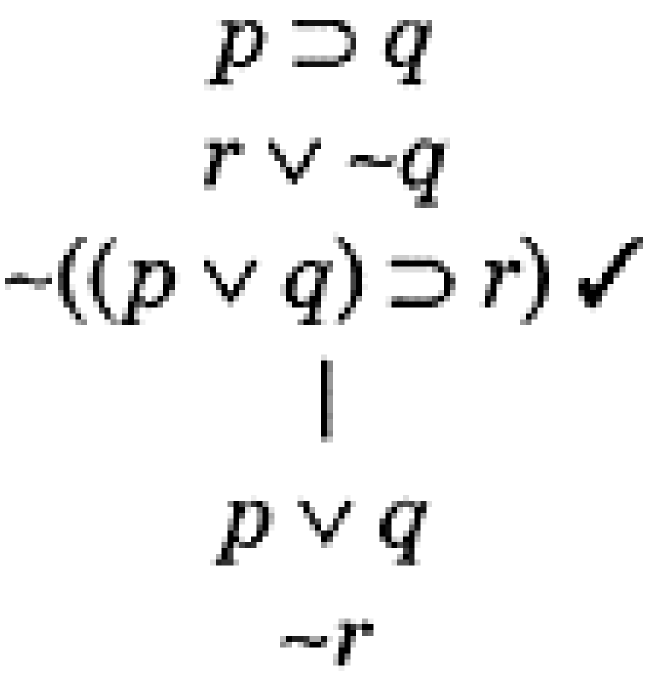
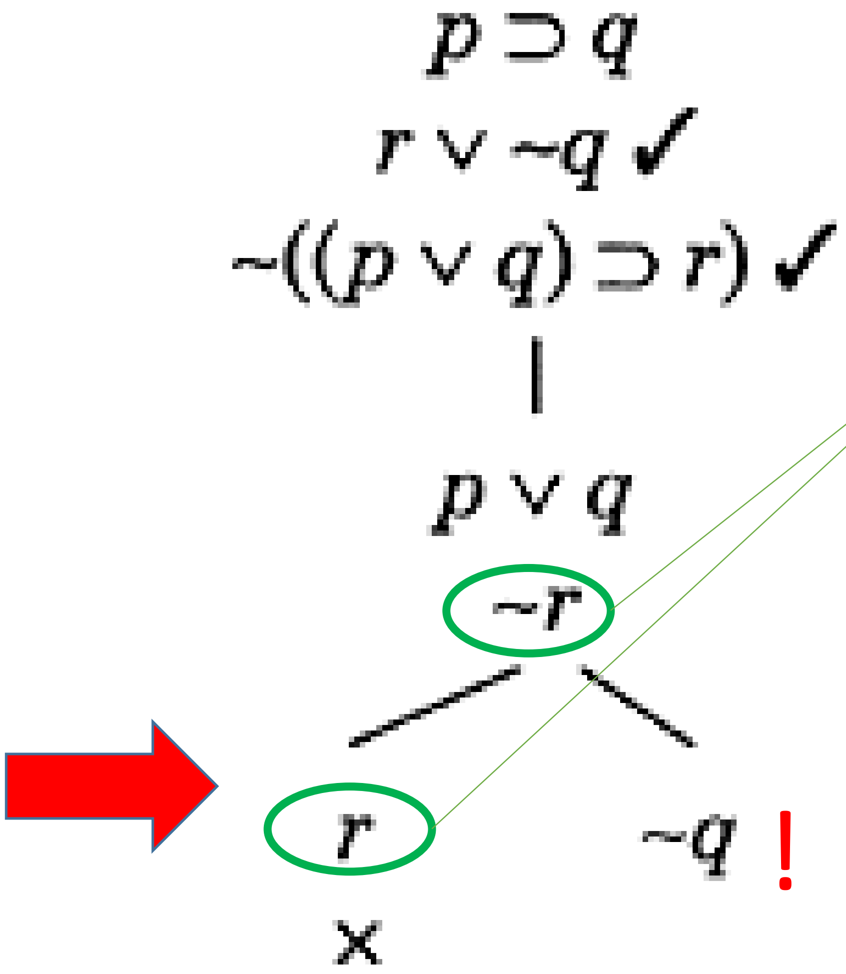


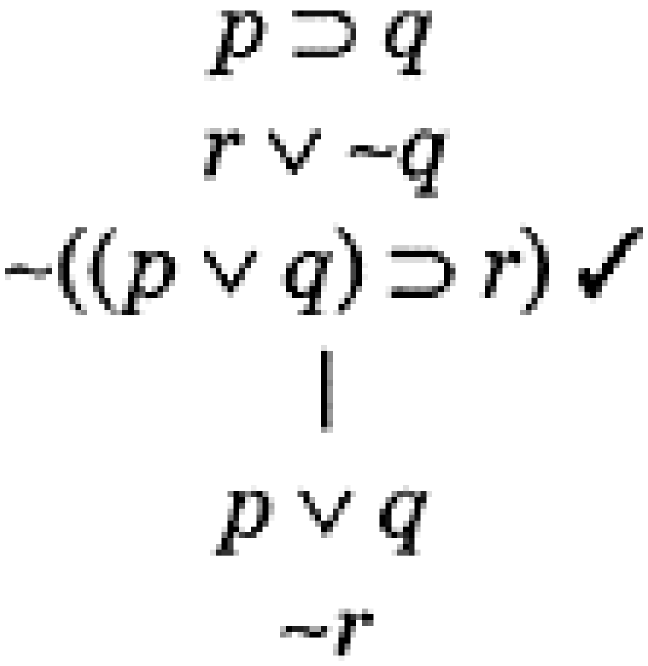


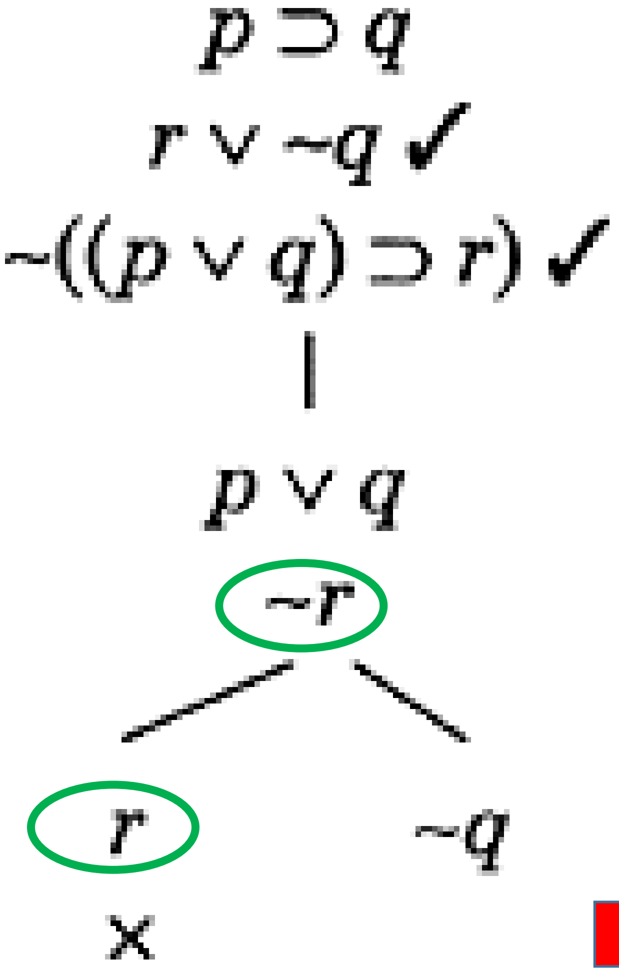
A vertical branch is closed (marked by an x) if there is a contradiction between propositions on it.

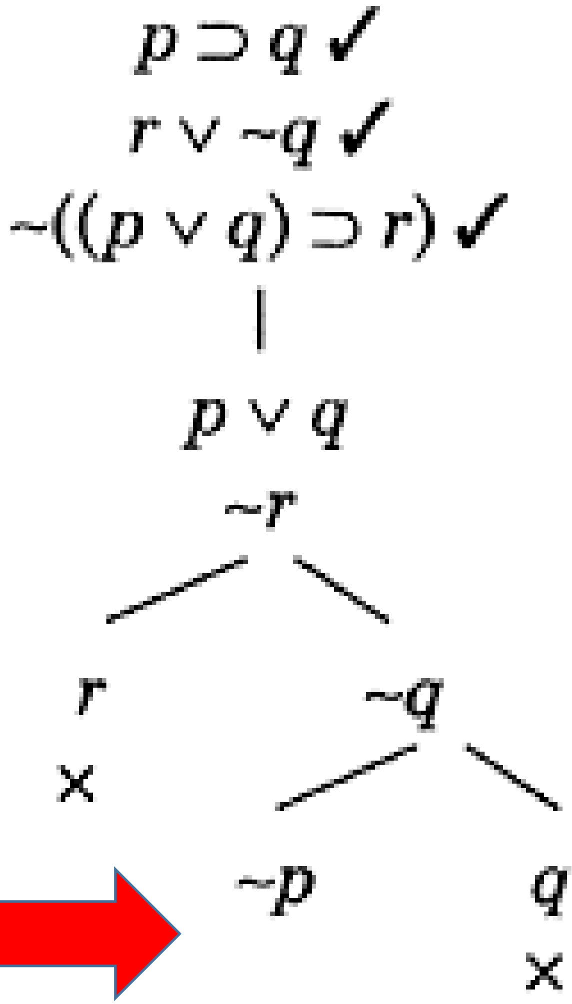
Analytic tables

(1): p⊃q
(2): rv¬q
C: ((pvq) ⊃r)

(1): p⊃q
(2): rv¬q
¬((pvq) ⊃r)







Analytic tables

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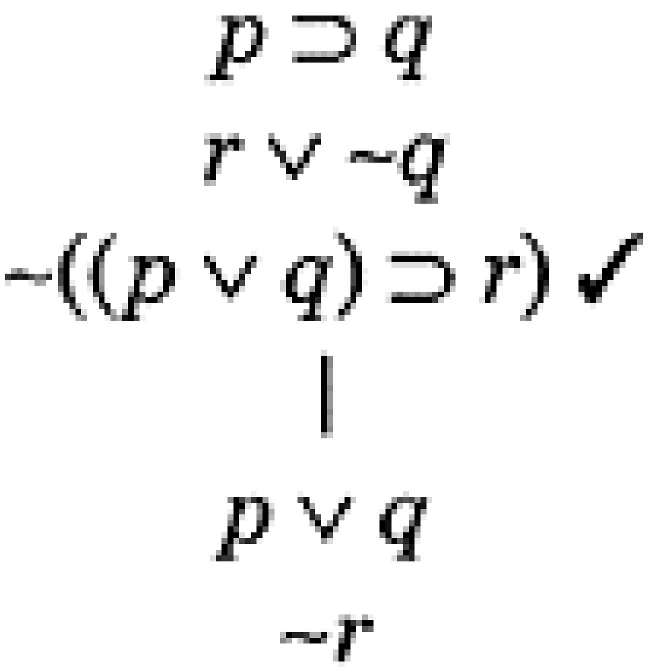

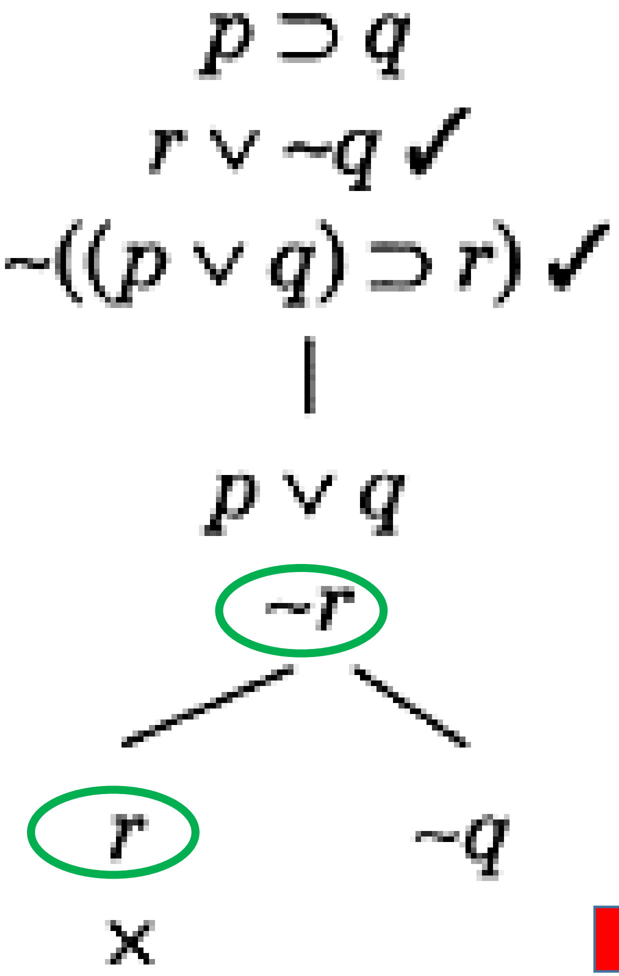

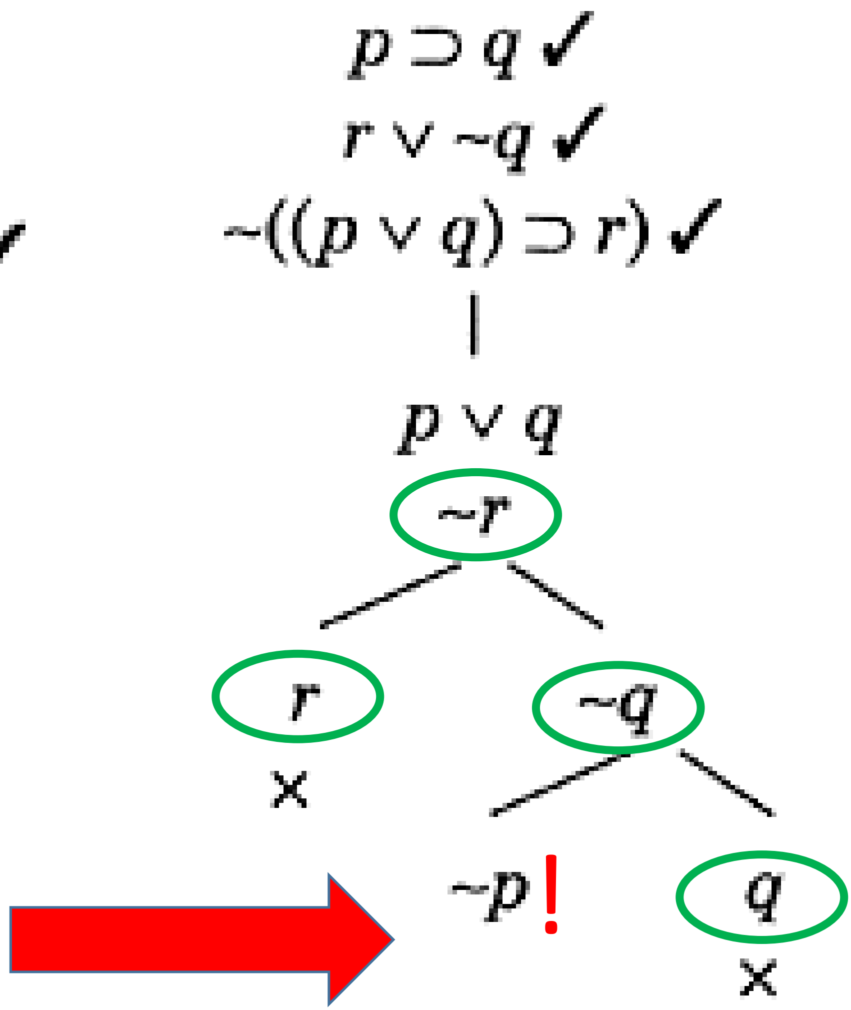

Analytic tables

(1): p⊃q
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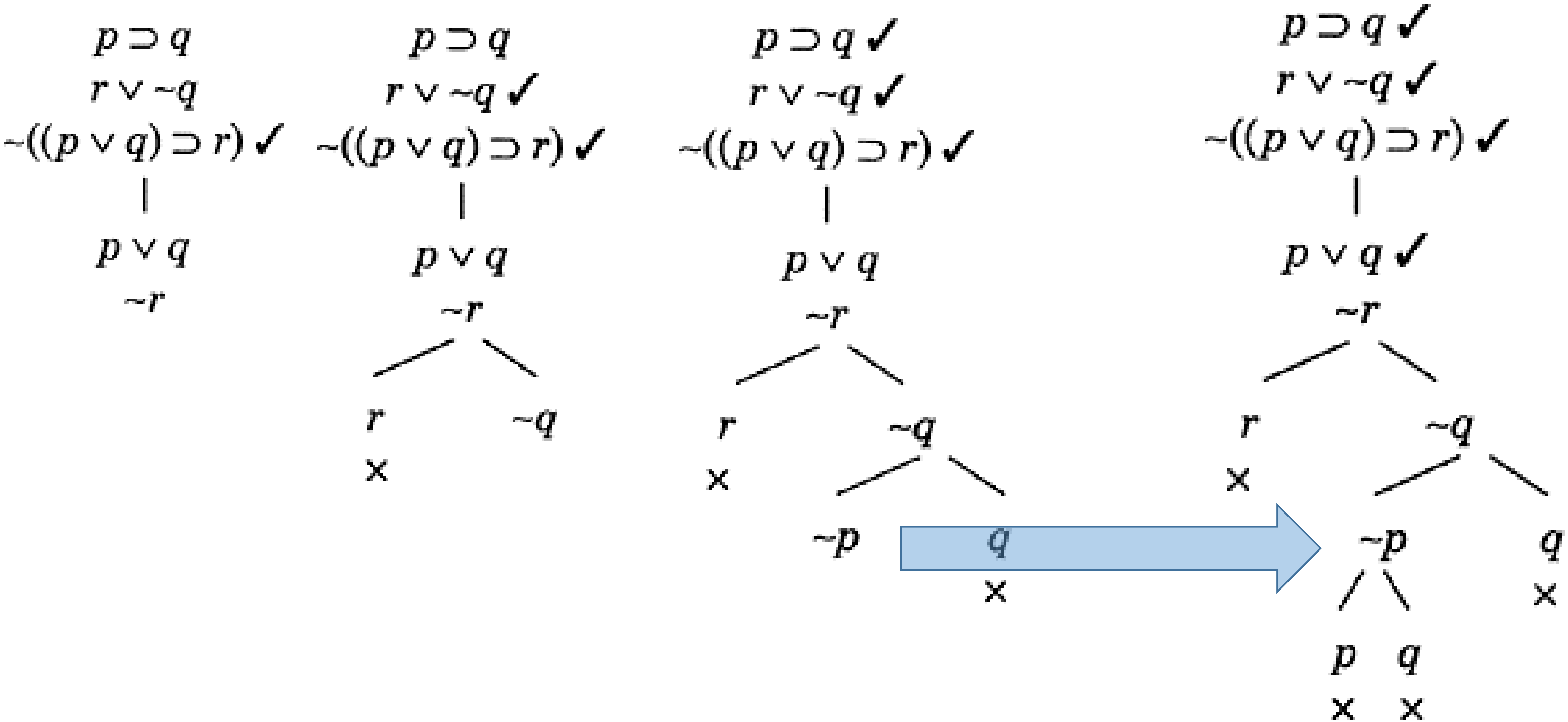
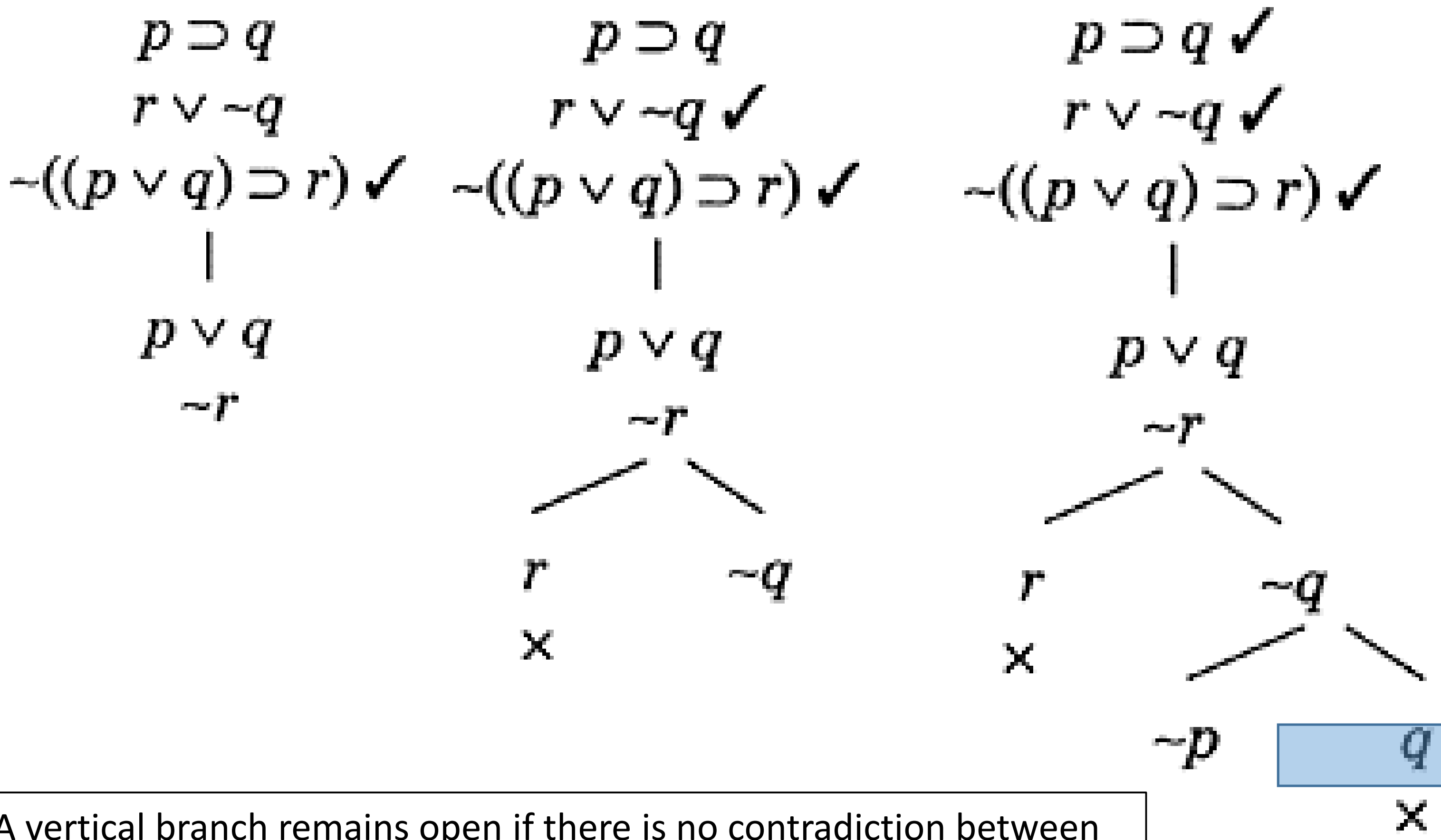
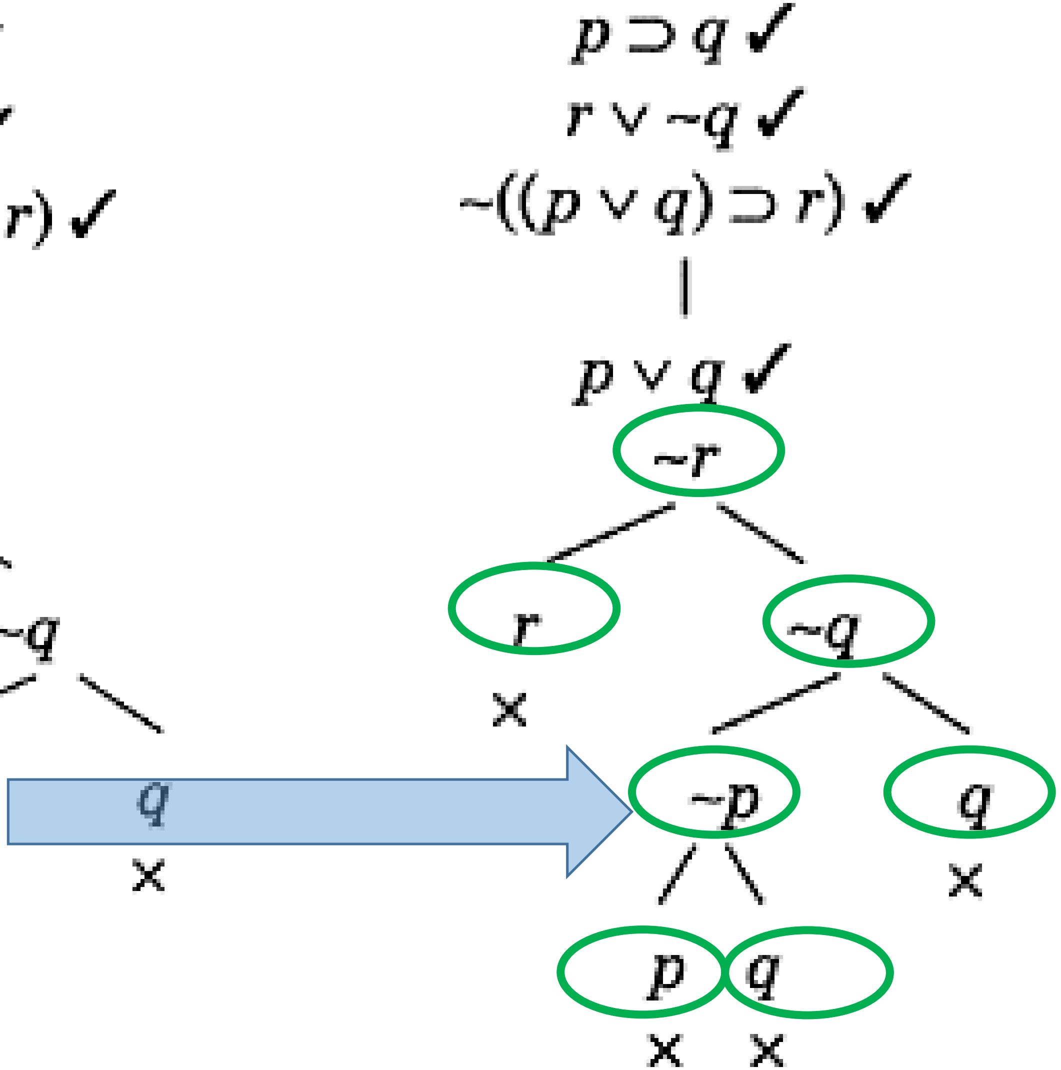


Analytic tables

(1): p⊃q
(2): rv¬q
C: ((pvq) ⊃r)

(1): p⊃q
(2): rv¬q
((¬pvq) ⊃r)





A vertical branch remains open if there is no contradiction between propositions on it after processing all propositions of the argument.
An argument without open branches is always valid.
An argument with open branches is never valid.

Inductive arguments, probability and strength

Reviewing the definition of arguments

Argument:

- “An argument is a list of statements, one of which is designated as the conclusion and the rest of which are designated as premises.”
- The conclusion states the point being argued for and the premises state the reasons being advanced in support of the conclusion.
- The standard logical form of an argument:
    - The list of premises
    - A line drawn under them
    - The conclusion under the line

An argument

Diodorus did not know how to tie a square knot.

All Eagle Scouts know how to tie square knots.

Diodorus was not an Eagle Scout.

Validity and strength

An argument is **deductively valid** if and only if it is **impossible** that its conclusion is false while its premises are true.

If an argument is deductively valid, its conclusion makes no claim that is not, at least implicitly, made by its premises.

An argument is **inductively strong** if and only if it is **improbable** that its conclusion is false given that its premises are true, and **it is not deductively valid**. The degree of inductive strength depends on how improbable it is that the conclusion is false while the premises are true.

If an argument is inductively strong, its conclusion makes factual claims that go beyond the factual information given in the premises.

Is this argument deductive or inductive?

George is a man.

George is 100 years old.

George has arthritis.

George will not run a four-minute mile tomorrow.

Inductive probability and strong inductive arguments

1 . The **inductive probability** of an argument is **the probability** that its conclusion is true given that its premises are true.

2. The inductive probability of an argument is **determined** by the evidential relation between its premises and its conclusion, not by the likelihood of the truth of its premises alone or the likelihood of the truth of its conclusion alone.

3. An argument is inductively strong if and only if:

a. Its inductive probability is high.

b. It is not deductively valid.

Credibility scale of argument structure

Deductively valid | Degrees of Inductive strength | Worthless